APERTURE AND F-NUMBER

DIGITAL MEDIA E-10: EXPOSING DIGITAL PHOTOGRAPHY
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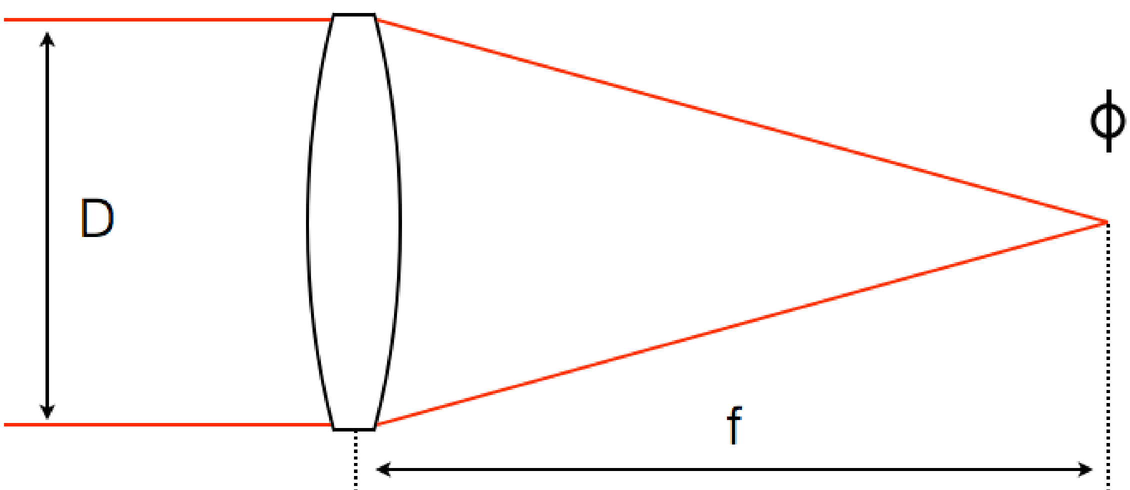
THE MATH BEHIND THE NUMBER

Electromagnetic radiation is made up of photons, elementary particles that exhibit properties not only of particles but also waves. Acting as a wave, a photon travels through space at the speed of light with a specific frequency that defines its energy and, therefore, its classification along the electromagnetic spectrum. Our eyes can only detect a very specific frequency range that makes up the visible spectrum (recall Roy G. Biv!). On the other hand, a photon acts as a particle such that the number of photons determines the intensity of light of a particular object emitting or reflecting those photons. Twice as many photons will appear twice as bright, or in photographic terms, "one stop" brighter.

Shutter speed is somewhat easy to understand in these terms. Doubling the time the shutter is open (say, from $\frac{1}{2}$ second to 1 second) doubles the number of photons that sensor can collect and consequently doubles the intensity of detected light. The F-number (also known as f-stop or aperture value) does not quite follow this pattern of doubling; it is a factor of about 1.4, rather than 2, that differentiates stops in the F-number.

A camera lens contains a diaphragm that restricts the amount of light reaching the film plane in a manner similar to the iris of the human eye. Whereas the hole that lets in light in the middle of the eyes' iris is called the pupil, the hole in the middle of a diaphragm of a lens is its aperture. Because an aperture is defined as a hole for light to pass through in an optical system, the pupil could also be called an aperture!



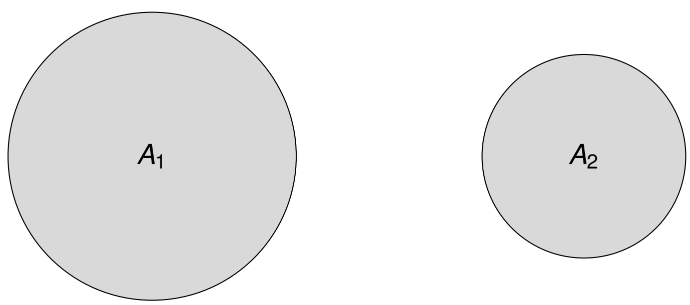
$$F - number = \frac{f}{D}$$

If the area of an aperture is doubled, the number of photons that can reach the sensor through the lens is also doubled. The F-number of a lens is the ratio of its focal length divided by the diameter of the aperture (Figure 1). Since the F-number is a ratio involving the diameter, and not the area, we lose the ability to nicely double or halve a number to calculate a stop.

Notice that, counterintuitively, higher F-numbers indicate a smaller aperture and therefore an increasing restriction on the light traveling through the lens. To convince yourself of this, imagine what would happen to the F-number if the aperture diameter decreases while the focal length remains the same or how it changes when the focal length increases while the diameter is held constant. Also realize that Figure 1, above, is an oversimplification of lenses. Almost all modern lenses are made up of more than one glass element, and it is not necessarily the very front element that contains the smallest diameter. An element inside of the lens may be a little bit smaller, for example, but this does not affect the focal length.

Although doubling the area of the aperture doubles the amount of light traveling through the lens, the F-number ratio is defined in terms of its area. The question therefore arises: what, precisely, constitutes a one stop difference in F-number?

To answer this question, let's assume we have two separate lenses that have maximum apertures with areas A_1 and A_2, as shown below.



A_1 is twice the area of A_2. For simplicity, let's assume that the diameter of A_1 (which we'll call d_1) and its focal length (f) are both 1, so the F-number of this lens would be 1. Let us further assume that the focal length of the lenses are the same. To figure out the F-number of A_2, we need to determine how much smaller the diameter is compared to that of A_1. Let's do some math!

$$\text{Area of a circle} = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 \qquad \text{radius } (r) = \frac{\text{diameter } (d)}{2}$$

$$\begin{aligned}
A_1 &= \pi \left(\frac{d_1}{2}\right)^2 = \pi \left(\frac{(d_1)^2}{4}\right) = \frac{\pi (d_1)^2}{4} \\
A_2 &= \pi \left(\frac{d_2}{2}\right)^2 = \pi \left(\frac{(d_2)^2}{4}\right) = \frac{\pi (d_2)^2}{4} \\
A_1 &= 2 \cdot A_2 \\
\frac{\pi (d_1)^2}{4} &= 2 \cdot \frac{\pi (d_2)^2}{4} \\
\frac{\frac{\pi (d_1)^2}{4}}{\left(\frac{\pi}{4}\right)} &= \frac{2 \cdot \frac{\pi (d_2)^2}{4}}{\left(\frac{\pi}{4}\right)} \\
(d_1)^2 &= 2(d_2)^2
\end{aligned}$$

$$
\begin{aligned}
d_2 &= \sqrt{\frac{1}{2}(d_1)^2} \\
&= \frac{\sqrt{1}}{\sqrt{2}} d_1 \\
&= \left(\frac{\sqrt{2}}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} d_1 \\
&= \frac{d_1\sqrt{2}}{2}
\end{aligned}
$$

This means, for A_2 to be half the area of A_1, the diameter d_2 must be $\frac{\sqrt{2}}{2}$ times d_1. The diameter d_2 is therefore computed as follows:

$$d_2 = \frac{d_1\sqrt{2}}{2} = \frac{1\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

Which is approximately 0.7. This demonstrates our contention that halving the area does not also halve its diameter. Let's continue with the calculations.

$$\text{F-number of } A_2 = \frac{f}{D} = \frac{f}{d_2} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \sqrt{2} \approx 1.4$$

The F-number for A_2, then, is $f/1.4$!

As an exercise, try doing the same thing with A_2 and a third circle, A_3, whose area is half of that of A_2. If you keep applying this math for halved areas, you get the list of F-numbers that are one stop apart:

F-numbers: $f/1.0, f/1.4, f/2.0, f/2.8, f/4.0, f/5.6, f/8, f/11, f/16, f/22,$ *etc.*

These numbers can be difficult to memorize; one approximation is to remember the first two F-numbers, 1.0 and 1.4, and double each to obtain the subsequent two F-numbers. You then double those to receive the following two, and so on. Like so:

2x 2x 2x 2x

F-numbers: $f/1.0, f/1.4, f/2.0, f/2.8, f/4.0, f/5.6, f/8.0, f/11, f/16, f/22,$ *etc.*

2x 2x 2x 2x

Please note, however, that this trick breaks down after about $f/32$ or so. The most accurate method for determining the sequence of F-numbers is to begin at $f/1.0$ and continuously multiply by $\sqrt{2}$, but this approximation is fine for nearly all circumstances since the typical range for F-numbers is below $f/32$.

Diameter of the Pinhole

It is possible to calculate the diameter of an aperture, including that of a pinhole, using the F-number ratio. In order to calculate the diameter we must know two values: the F-number and the focal length.

The focal length is easy to calculate by measuring the distance perpendicular from the focal plane to the end of the pinhole. The location of the film plane on an SLR is indicated by the focal plane icon printed or embossed on the body.

In the case of the pinhole used in class, the focal length is 50mm.

Next, let's calculate the F-number of the pinhole by determining how many stops darker it is compared to a lens with a known F-number. We'll set up the camera on a tripod and take a properly exposed photo at ISO 100 and some shutter speed. We then will take photos using the pinhole lens at a variety of shutter speeds to identify one whose exposure matches that of the original photo taken with the lens.

For this calculation, the properly exposed photo was $\frac{1}{1250}$ of a second at $f/1.4$ and ISO 100. After a few attempts at the same ISO we measured the pinhole exposure at 4 seconds. Dividing 4 by $\frac{1}{1250}$ shows that the pinhole image required a shutter speed 5,000 times longer than that of the lens! Stops are measured in doubling or halving of light so we must figure out how many times 2 is multiplied with itself to reach 5,000:

$$
\begin{aligned}
2^x &= 5000 \\
\log 2^x &= \log 5000 \\
x \log 2 &= \log 5000 \\
x &= \frac{\log 5000}{\log 2} = \log_2 5000 \approx 12.3
\end{aligned}
$$

The light was halved roughly 12 times. From this, we can determine the F-number of the pinhole by counting 12 stops away from $f/1.4$:

F-numbers: $f/1.4$, $f/2$, $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, $f/16$, $f/22$, $f/32$, $f/45$, $f/64$, **f/90**

As a reminder, our trick to double F-numbers from $f/1.0$ and $f/1.4$ is only accurate up to a point. The above reflects the true F-number scale beyond $f/32$.

With this we can now perform our final calculation!

$$
\begin{aligned}
\text{F-number} &= \frac{f}{D} \\
90 &= \frac{50\text{mm}}{D} \\
D &= \frac{50\text{mm}}{90} = 0.5\overline{6} \approx \mathbf{0.5mm}
\end{aligned}
$$

Because the aperture is actually 12.3 stops darker, and not just 12, the true F-number is slightly more than $f/90$, and we therefore round down the above calculation for our final result of $0.5mm$.